Momentum

Chapter 9

What you already know:

- **Velocity**:
  - A vector quantity that is a measure of the change in displacement per unit change in time.
- **Acceleration**:
  - A vector quantity that is a measure of the change in velocity per unit change in time.
- Mass:
  - A scalar quantity that is a measure of the amount of matter an object contains.
- **Force**:
  - A vector quantity consisting of a push or pull that may cause an object to change direction or velocity, or both.

Momentum (**p**)

- What is momentum?
  - Momentum is a **vector quantity** that is the product of an object's mass times its velocity.

$$\mathbf{p} = m\mathbf{v}$$

    - Momentum can be thought of as the tendency of an object to continue to move in a direction of travel.
    - Momentum can be thought of as mass in motion.

Basic concepts

- Conservation of Momentum: Momentum is conserved in any collision between objects
- $\mathbf{p}_i = m_i\mathbf{v}_i = m_f\mathbf{v}_f = \mathbf{p}_f$ (Initial Momentum equals final)
- $\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$ If two bodies interact, initial sum equals final sum
- Since **v** is a vector, it may be broken down into components as appropriate

Effects of mass and velocity on Momentum

- A bowler is experimenting with a couple of bowling balls, one with a mass of 3.5 kg and the other with a mass of 7.0 kg.
  - What will be the effect on momentum if the bowler changes from the 3.5 kg bowling ball to the 7.0 kg bowling ball *if the velocity remains constant*?
  - What will be the effect on momentum if the bowler changes the velocity with which he bowls from 1 m/s to 2 m/s?
  - Which one results in greater energy?
- Changes in mass and velocity are directly proportional to changes in momentum; e.g. if you double one, you will double the other.

Impulse

- How do you stop an object from moving?
  - You apply a force.
    - If the force is applied in the opposite direction, it will slow the object down.
    - If the force is applied in the same direction, it will cause the object to speed up.
  - Impulse: $\mathbf{J} = \mathbf{F}_{\mathbf{net}}\Delta t$
  - Impulse is a **vector** quantity

Impulse and Momentum

- Let's start with
  - $\mathbf{V_f = v_i + at}$

  with which we are all comfortable

- Multiply through by mass m to get
  - $Mv_f = mv_i + mat = mv_i + Ft \rightarrow$
      - $p_f = p_i + Ft$, or
  - This says that if there is no force, momentum is conserved. To get a change in momentum, apply a force.

Impulse and Momentum

- We also express
  - $p_f = p_i + Ft$

  as

  - $J = Ft = \Delta p = p_f - p_i$

Where J is called impulse.

This is called the impulse – momentum theorem. The impulse **($F_{net}\Delta t$)** is equal to the ***change*** in momentum (Δp) that the force causes.

Impulse and Newton's 2nd Law

- Newton's 2nd Law of Motion:

$$\mathbf{F}_{\mathbf{net}} = m\mathbf{a} = m\frac{\Delta \mathbf{v}}{\Delta t}$$

  - If you multiply both sides by Δt

$$\mathbf{F}_{\mathbf{net}}\Delta t = m\Delta \boldsymbol{v} = m\mathbf{v}_f - m\mathbf{v}_i$$

  - or

$$\mathbf{F}_{\mathbf{net}}\Delta t = \mathbf{p}_f - \mathbf{p}_i$$

  - **This equation is the impulse-momentum theorem.**
    - **The impulse ($F_{net}\Delta t$) is equal to the *change* in momentum (Δp) that the force causes.**

Skip

Units for Impulse and Momentum

- What are the units for momentum?

1 Unit of Momentum = 1 kg•m/s

- What are the units for Impulse?

1 Unit of Impulse = 1 N•s

- Since impulse equals momentum:

1 N•s = 1 kg•m/s

Example 1:

- A batter makes contact with a 0.145 kg baseball traveling at 40 m/s with an average force of 5,000 N for 0.003 seconds. What is the momentum and velocity of the ball after it leaves the bat.

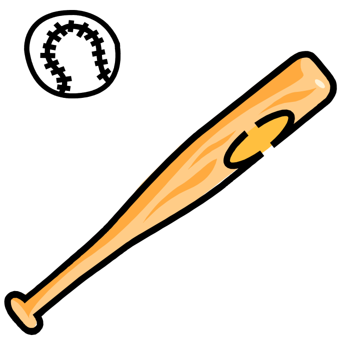
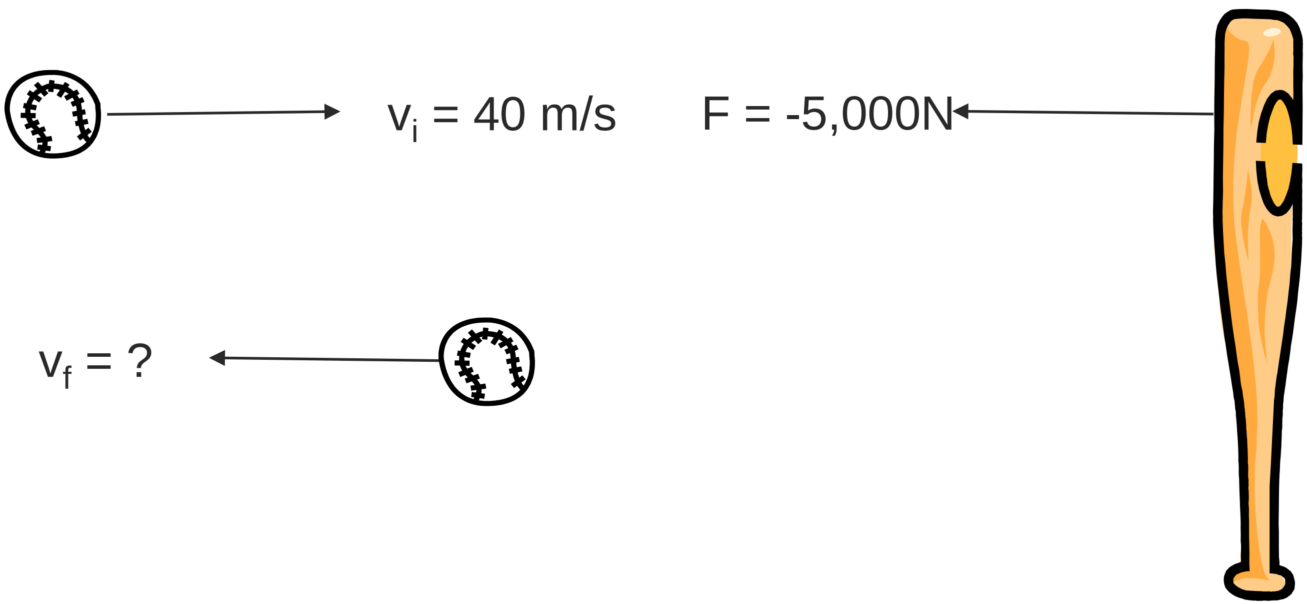


Diagram the Problem

- If the initial velocity of the ball is assumed to be in the positive direction, then the ball will be moving in the negative direction after making contact with the bat.



Solve the Problem

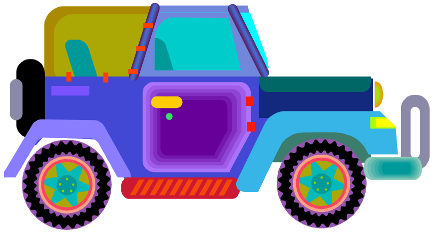
- $\mathbf{J} = \mathbf{F}_{\mathbf{net}}\Delta t = \mathbf{p}_f - \mathbf{p}_i$
- $\mathbf{J} = \mathbf{F}_{\mathbf{net}}\Delta t = m\boldsymbol{v}_f - m\boldsymbol{v}_i$
- $m\boldsymbol{v}_f = \mathbf{F}_{\mathbf{net}}\Delta t + m\boldsymbol{v}_i$
- $m\boldsymbol{v}_f = (-5{,}000\text{N})(0.003\text{s}) + (0.145\text{kg})(40\text{m/s})$
- $\mathbf{p}_f = -9.2\ \text{kg}\bullet\text{m/s}$
- $\boldsymbol{v}_f = \mathbf{p}_f/m = (-9.2\ \text{kg}\bullet\text{m/s})/(0.145\text{kg})$
- $\boldsymbol{v}_f = -63\ \text{m/s}$

Using Impulse and Momentum for Safety

- A large impulse will result in a large change in momentum.
  - A large impulse can result from a large force over a very short period of time.
  - A large impulse can result from a smaller force over an extended period of time.
- For automotive safety, reduces the forces on the occupants by extending the time over which deceleration occurs.

Example 2:

- A 2,200 kg SUV is traveling at 94 km/hr (~55 mph) stops in 21 seconds when using the brakes gently or 5.5 seconds when in a panic. However, the vehicle will come to a halt in 0.22 seconds if it hits a concrete wall. What is the average force exerted in each of these stops?



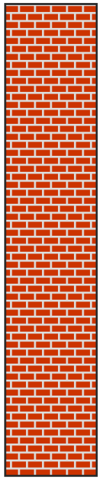
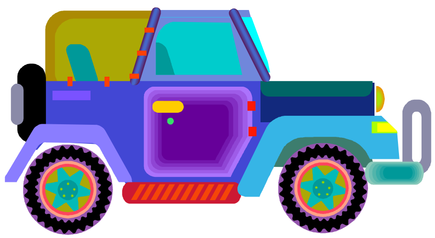
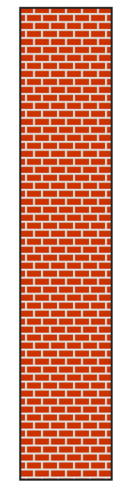


Diagram the Problem





$\boldsymbol{v}_i$ = 94 km/hr = 26 m/s

$\boldsymbol{v}_f$ = 0 m/s

$\mathbf{p}_i = m\boldsymbol{v}_i = 5.72 \times 10^4$ kg•m/s

$\mathbf{p}_f$ = 0 kg•m/s

Impulse (**F**Δt)

Solve the Problem

- $\mathbf{J} = \mathbf{F}\,\Delta t = \mathbf{p}_f - \mathbf{p}_i$
- $\mathbf{F}\,\Delta t = m\cancelto{0}{\boldsymbol{v}_f} - m\boldsymbol{v}_i$
- $\mathbf{F}\,\Delta t = -m\boldsymbol{v}_i$
- $\mathbf{F} = -m\boldsymbol{v}_i/\Delta t$

t	21 s	5.5 s	0.22 s
F	-2,700 N (607 lbs)	-10,000 N (2,250 lbs)	-260,000 N (58,400 lbs)

Collisions

- Two types
  - Elastic collisions – objects may deform but after the collision end up unchanged
    - Objects separate after the collision
    - Example: Billiard balls
    - Kinetic energy is conserved (no loss to internal energy or heat)
  - Inelastic collisions – objects permanently deform and / or stick together after collision
    - Kinetic energy is transformed into internal energy or heat
    - Examples: Spitballs, railroad cars, automobile accident

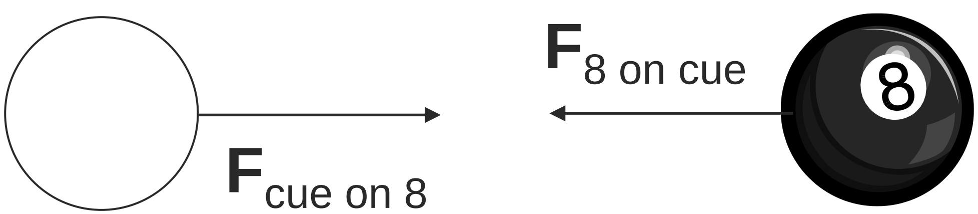
Example 3:

- An asteroid (d=270m, m = 4E10 kg) was initially determined to be on a course that could possibly lead to collision with the Earth in 2036 after passing close by in 2029. Assuming an ion engine of thrust 0.5N is attached to the asteroid in 2029, what change in velocity could be given to the asteroid after a year of firing the engine?
- Would it be enough to move the asteroid out of a collision course in the next 6 years?

- Note: not done in 2016 or 2017

Conservation of Momentum

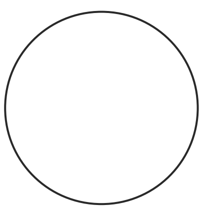
- Newton's 3rd Law of motion says that for every action there is an equal and opposite reaction.
  - The force on one object is equal and opposite the force on the other object



Is this useful for Regents students?

Collisions

- Assume both balls are moving in opposite directions.
- The Impulse-Momentum Theorem can be used to analyze the collision from both object's perspective
  - For cue ball: $\mathbf{F}_{8\text{ on cue}}\Delta t = \mathbf{p}_{cue(f)} - \mathbf{p}_{cue(i)}$ (1)
  - For 8 ball: $\mathbf{F}_{\text{cue on }8}\Delta t = \mathbf{p}_{8(f)} - \mathbf{p}_{8(i)}$ (2)



Collisions

- Solving (1) and (2) for the initial momentum of each object before the collision gives us:
  - $\mathbf{p}_{cue(i)} = \mathbf{p}_{cue(f)} - \mathbf{F}_{8\text{ on }cue}\Delta t$ (3)
  - $\mathbf{p}_{8(i)} = \mathbf{p}_{8(f)} - \mathbf{F}_{cue\text{ on }8}\Delta t$ (4)
- As per Newton's 3rd Law: $\mathbf{F}_{cue\text{ on }8} = -\mathbf{F}_{8\text{ on }cue}$
  - Substituting the latter into (4) and then adding the two equations together yields:
    - $\mathbf{p}_{cue(i)} = \mathbf{p}_{cue(f)} - \mathbf{F}_{8\text{ on }cue}\Delta t$
    - $\mathbf{p}_{8(i)} = \mathbf{p}_{8(f)} + \mathbf{F}_{8\text{ on }cue}\Delta t$
    - $\mathbf{p}_{cue(i)} + \mathbf{p}_{8(i)} = \mathbf{p}_{cue(f)} + \mathbf{p}_{8(f)}$

Law of Conservation of Momentum

- Hence, the sum of the momenta of two bodies before a collision is the same as the sum of their momenta after a collision.

$$\mathbf{p}_{1(i)} + \mathbf{p}_{2(i)} = \mathbf{p}_{1(f)} + \mathbf{p}_{2(f)}$$

or

$$m_1\boldsymbol{v}_{1(i)} + m_2\boldsymbol{v}_{2(i)} = m_1\boldsymbol{v}_{1(f)} + m_2\boldsymbol{v}_{2(f)}$$

- It is most simply written as:

$$p_{before} = p_{after}$$

- Conservation of Momentum is true for a closed system where all the forces are internal.

Example 4

- **Cart A approaches cart B, which is initially at rest, with an initial velocity of 30 m/s. After the collision, cart A stops and cart B continues on with what velocity? Cart A has a mass of 50 kg while cart B has a mass of 100kg.**

A

B

Diagram the Problem

A

B

Before Collision:

$\mathbf{p}_{A1} = m\boldsymbol{v}_{A1}$ $\longrightarrow$

$\mathbf{p}_{B1} = m\boldsymbol{v}_{B1} = 0$

After Collision:

$\mathbf{p}_{A2} = m\boldsymbol{v}_{A2} = 0$

$\mathbf{p}_{B2} = m\boldsymbol{v}_{B2}$ $\longrightarrow$

Solve the Problem

- $\mathbf{p}_{before} = \mathbf{p}_{after}$
- $m_A\boldsymbol{v}_{A1} + \cancelto{0}{m_B\boldsymbol{v}_{B1}} = \cancelto{0}{m_A\boldsymbol{v}_{A2}} + m_B\boldsymbol{v}_{B2}$
- $m_A\boldsymbol{v}_{A1} = m_B\boldsymbol{v}_{B2}$
- $(50\text{ kg})(30\text{ m/s}) = (100\text{ kg})(\boldsymbol{v}_{B2})$
- $\boldsymbol{v}_{B2} = 15\text{ m/s}$
- Is kinetic energy conserved?

Example 5

Per 7

- **Cart A approaches cart B, which is initially at rest, with an initial velocity of 30 m/s. After the collision, cart A and cart B continue on together with what velocity? Cart A has a mass of 50 kg while cart B has a mass of 100kg.**

Diagram the Problem

A **B**

Before Collision:

$\mathbf{p}_{A1} = m\boldsymbol{v}_{A1} \longrightarrow$

$\mathbf{p}_{B1} = m\boldsymbol{v}_{B1} = 0$

After Collision:

$\mathbf{p}_{A2} = m\boldsymbol{v}_{A2} \longrightarrow$

$\mathbf{p}_{B2} = m\boldsymbol{v}_{B2} \longrightarrow$

Note: Since the carts stick together after the collision, $\boldsymbol{v}_{A2} = \boldsymbol{v}_{B2} = \boldsymbol{v}_2$.

Solve the Problem

- $\mathbf{p}_{before} = \mathbf{p}_{after}$
- $m_A\boldsymbol{v}_{A1} + m_B\cancelto{0}{\boldsymbol{v}_{B1}} = m_A\boldsymbol{v}_{A2} + m_B\boldsymbol{v}_{B2}$
- $m_A\boldsymbol{v}_{A1} = (m_A + m_B)\boldsymbol{v}_2$
- $(50\text{ kg})(30\text{ m/s}) = (50\text{ kg} + 100\text{ kg})(\boldsymbol{v}_2)$
- $\boldsymbol{v}_2 = 10\text{ m/s}$
- Is kinetic energy conserved?

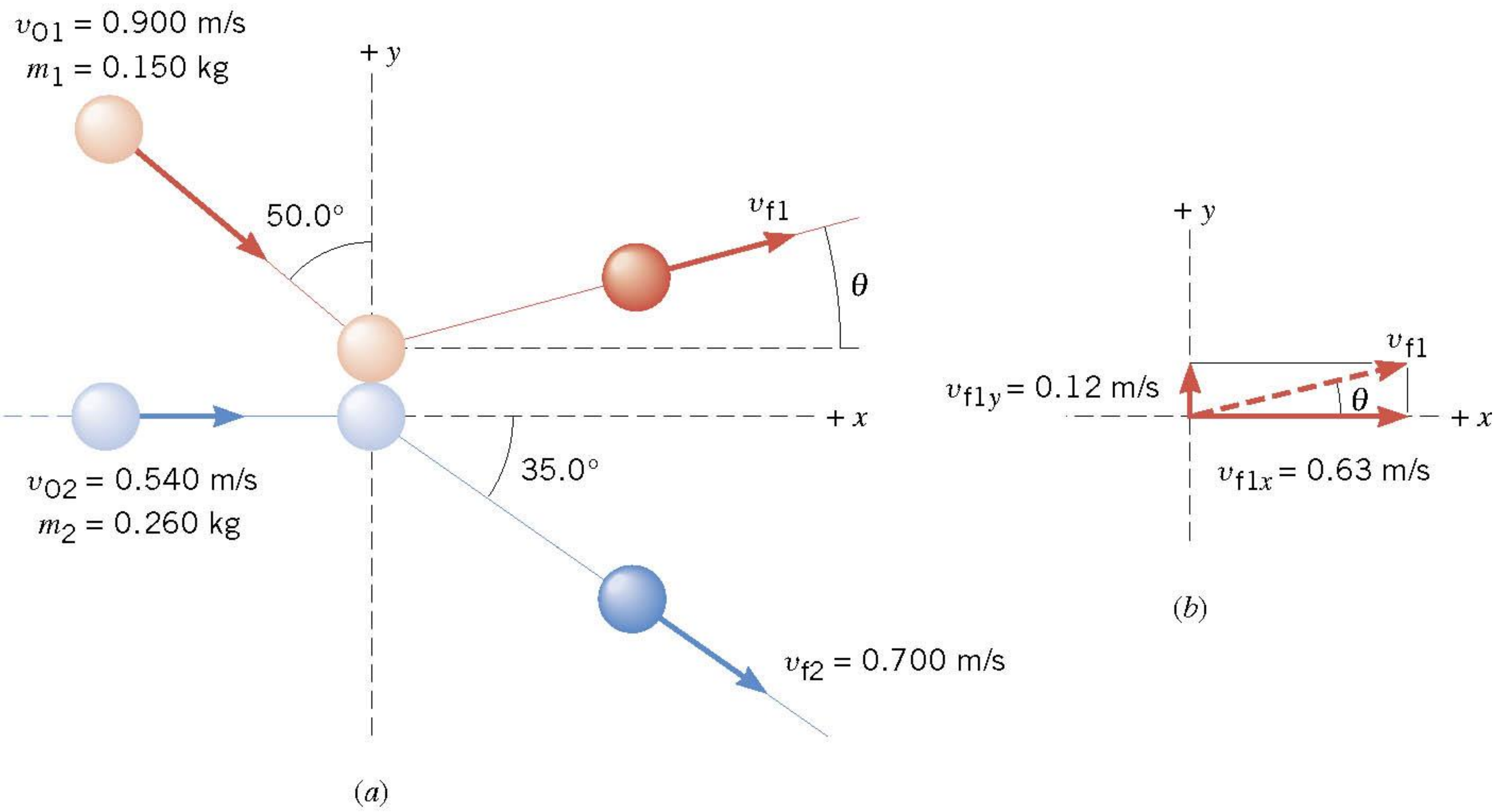
Key Ideas

- Momentum is a vector quantity equal to the mass of an object times its velocity.
- Impulse is equal to the force on an object times the amount of time that the force was applied to the object.
- The impulse momentum theorem equates impulse to momentum ($F\Delta t = m\Delta v$).
- Conservation of momentum requires that the momentum of a system before a collision is equal to the momentum of the system after the collision.

Movie

- http://www.newtonsapple.tv/video.php?id=902
- https://video.search.yahoo.com/video/play;_ylt=A0LEVv6M2rdUxz0AYholnIlQ;_ylu=X3oDMTBsa3ZzMnBvBHNlYwNzYwRjb2xvA2JmMQR2dGlkAw--?p=impulse+and+momentum+air+bags&tnr=21&vid=C22C22529497B9F5271CC22C22529497B9F5271C&l=124&turl=http%3A%2F%2Fts1.mm.bing.net%2Fth%3Fid%3DUN.608018463030248476%26pid%3D15.1&rurl=http%3A%2F%2Fwww.youtube.com%2Fwatch%3Fv%3Dy118jLg20i0&sigr=11aosh50a&tt=b&tit=Crash+Test+Dummies+Seatbelts%

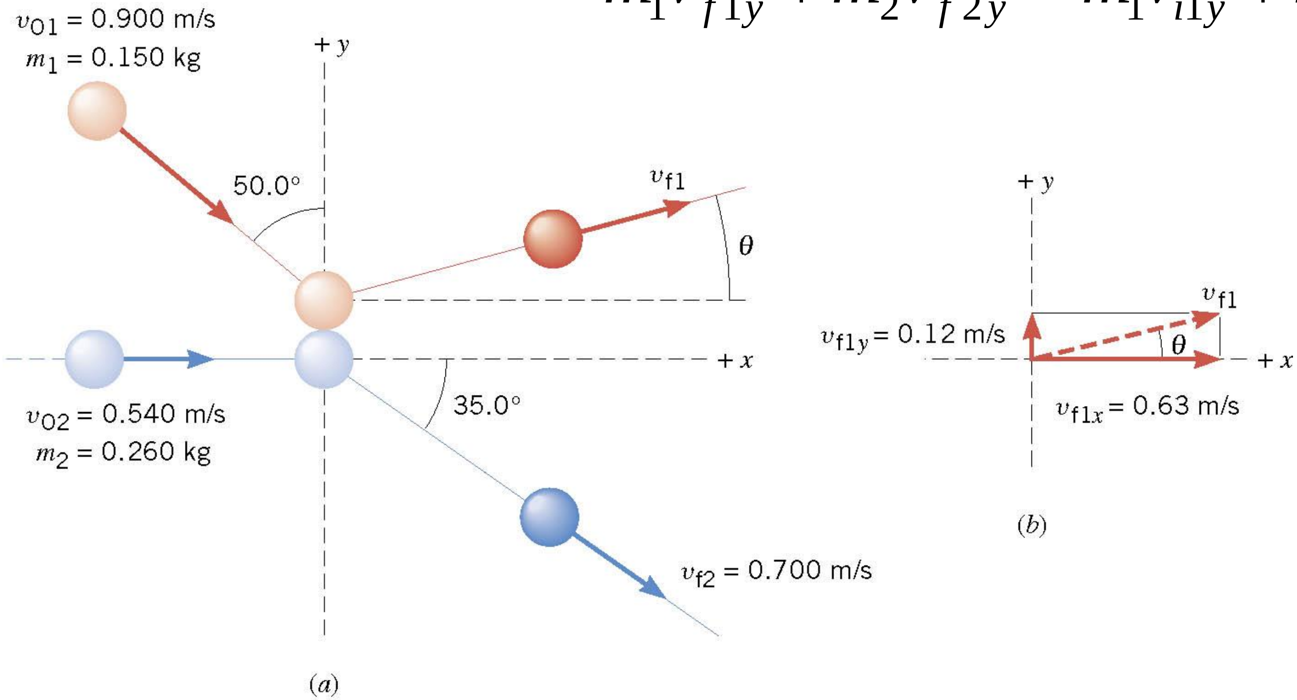
7.4 Collisions in Two Dimensions



7.4 Collisions in Two Dimensions

$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{i1x} + m_2 v_{i2x}$$

$$m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{i1y} + m_2 v_{i2y}$$

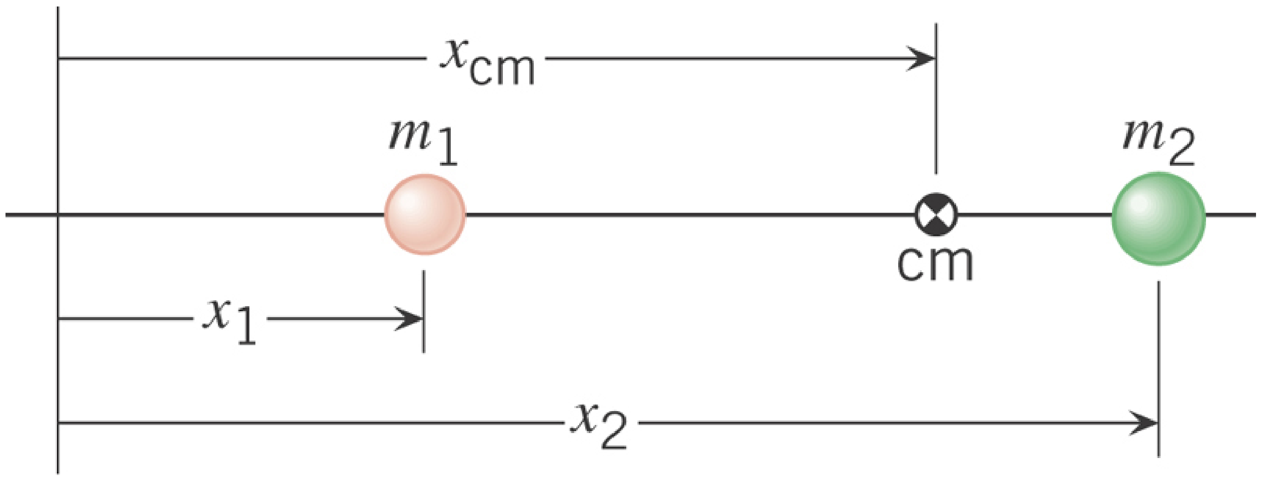


Not Covered

Center of Mass

- A measure of the average location for the total mass of a system of objects.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$



Center of Mass and Momentum

- While the velocity of various particles in a system may change in the event of a collision, the velocity of the center of mass will remain constant before and after the collision.

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2 + \ldots}{m_1 + m_2 + \ldots}$$